Spring 2014 **Math 601** Name: ____________________

Quiz 6

Question 1. (6 pts)
Determine whether each of the following statements is true or false. You do NOT need to explain.

(a) If S is an orthogonal set of nonzero vectors, then S is linearly independent.

Solution: True.

(b) For an inner product vector space V of dimension n, we can always find an orthonormal set of n vectors.

Solution: True.

(c) A square matrix is orthogonal if and only if its column vectors form an orthonormal set.

Solution: True.

Question 2. (4 pts)
Let $u_1 = (1, 0, 2)$, $u_2 = (-2, 0, 1)$ and $u_3 = (0, 1, 0)$. We know that $S = \{u_1, u_2, u_3\}$ is an orthogonal basis of $\mathbb{R}^3$. Now suppose $v = (1, 1, 1)$. Find the coordinates of v with respect to the basis S.

Solution: Since u_1, u_2 and u_3 are orthogonal, we can use the formula

$$v = \frac{\langle v, u_1\rangle}{\langle u_1, u_1\rangle}u_1 + \frac{\langle v, u_2\rangle}{\langle u_2, u_2\rangle}u_2 + \frac{\langle v, u_3\rangle}{\langle u_3, u_3\rangle}u_3 = \frac{3}{5}u_1 + \frac{-1}{5}u_2 + u_3$$

It follows that

$$[v]_S = \begin{bmatrix} 3/5 \\ -1/5 \\ 1 \end{bmatrix}$$

Question 3. (10 pts)

Let U be the subspace of $\mathbb{R}^4$ spanned by $v_1 = (1, 7, 1, 7), v_2 = (0, 7, 2, 7)$ and $v_3 = (1, 8, 1, 6)$. Use the Gram-Schmidt process to find an *orthonormal* basis of U.

Solution: I will leave out the details.

$$w_1 = \frac{1}{10}(1, 7, 1, 7)$$

$$w_2 = \frac{1}{\sqrt{2}}(-1, 0, 1, 0)$$

$$w_3 = \frac{1}{\sqrt{2}}(0, 1, 0, -1)$$